Group - I

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) Minutes

(INTER PART-I) 319-(I)
Code: 6191
OBJECTIVE

GROUP: I

PAPER: I
Marks: 20

You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question. Attempt as many questions as given in objective type question paper and leave others blank.

The imaginary part of a complex number $\overline{a+ib}$ is
(A) $-b$ (B) b (C) a (D) $-a$

The converse of $p \rightarrow q$ is
(A) $\sim p \rightarrow q$ (B) $p \rightarrow \sim q$ (C) $q \rightarrow p$ (D) $\sim p \rightarrow \sim q$

A square matrix A is said to be Hermitian if $(\overline{A})^t$
(A) $\overline{A}$ (B) A^t (C) A (D) $-A$

The trivial solution of the homogeneous linear equation is
(A) (0, 0, 0) (B) (1, 0, 0) (C) (0, 1, 0) (D) (0, 0, 1)

5- Roots of $x^2 - x - 2 = 0$ are
(A) $2, -1$ (B) $-2, 1$ (C) $-2, -1$ (D) $2, 1$

6- If one solution of the equation $x^2 + ax + 2 = 0$ is $x = 1$, then $a =$
(A) -7 (B) 7 (C) -3 (D) 0

7- A fraction in which the degree of the numerator is less than the degree of the denominator is called
(A) a proper fraction (B) partial fraction (C) combined fraction (D) irrational fraction

8- The sequence 3, 6, 12, ……… is
(A) A.P (B) G.P (C) H.P (D) Arithmetic Series

9- $\sum_{k=1}^{n} k^2 =$
(A) $\frac{n(n+1)(2n+1)}{6}$ (B) $\frac{n(n+1)}{4}$ (C) $\frac{n(n+1)}{2}$ (D) $n(n+1)$

10- The factorial of a positive integer 'n' is
(A) $n! = n(n-1)!(n-2)!$ (B) $n! = n(n+2)!$
(C) $n! = n(n-1)!$ (D) $n! = n(n-2)!$

11- If A and B are two disjoint events, then $P(A \cup B) =$
(A) $P(A) + P(B)$ (B) $P(A) + P(B) - P(A \cup B)$
(C) $P(A) - P(B)$ (D) $P(A) + P(B) - P(A \cap B)$

(Turn over)

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12- If 'n' is positive integer, then $n^3 + n$ is divided by

(A) 2 (B) 3 (C) 4 (D) 5

13- The number of terms in the expansion $(x - 3)^{10}$ is

(A) 10 (B) 11 (C) 12 (D) 13

14- If $\cos\theta = \frac{1}{\sqrt{2}}$, then θ is equal to

(A) 30° (B) 45° (C) 60° (D) 90°

15- $\tan\frac{\alpha}{2} =$

(A) $\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$ (B) $\pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}$ (C) $\pm\sqrt{\frac{1-\cos\alpha}{2}}$ (D) $\pm\sqrt{\frac{1+\cos\alpha}{2}}$

16- The period of $3\sin\frac{x}{3}$ is

(A) π (B) 2π (C) 3π (D) 6π

17- $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$ is called the law of

(A) tangents (B) cosine (C) sines (D) cotangents

18- If Δ is the area of a triangle ABC then $\Delta =$

(A) $\frac{1}{2}bc\sin\beta$ (B) $\frac{1}{2}ab\sin\alpha$ (C) $\frac{1}{2}bc\sin\alpha$ (D) $ab\sin\alpha$

19- $\sec\left(\cos^{-1}\frac{1}{2}\right) =$

(A) $\frac{1}{2}$ (B) 2 (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{6}$

20- $\cos x = \frac{1}{2}$ has a solution

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

Group I

thematics (INTER PART-I) 319 GROUP: I PAPER: I

1e: 2:30 hours **SUBJECTIVE** Marks: 80

te: Section I is compulsory. Attempt any three (3) questions from Section II.

SECTION I

Write short answers to any EIGHT questions: (2 x 8 = 16)

i- Find modulus of $1 - i\sqrt{3}$

ii- Prove that sum as well as the product of any two conjugate complex numbers is a real number.

iii- Does the set $\{1, -1\}$ posses closure properties with respect to addition and multiplication?

iv- Define a binary relation from a set A to a set B.

v- Let $A = \{1, 2, 3\}$. Determine the relation r such that $x r y$ iff $x < y$.

vi- What is proposition?

vii- Define row and column matrices.

viii- Without expansion, verify that $\begin{vmatrix} \alpha & \beta+\gamma & 1 \\ \beta & \gamma+\alpha & 1 \\ \gamma & \alpha+\beta & 1 \end{vmatrix} = 0$

ix- If $A = \begin{vmatrix} 2 & -1 \\ 3 & 1 \end{vmatrix}$, verify that $(A^{-1})^t = (A^t)^{-1}$

x- Prove that $(-1+\sqrt{-3})^4 + (-1-\sqrt{-3})^4 = -16$

xi- If α, β are the roots of $3x^2 - 2x + 4 = 0$, then find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

xii- Show that $x - 2$ is a factor of $x^4 - 13x^2 + 36$

Write short answers to any EIGHT questions: (2 x 8 = 16)

i- Define improper rational fraction.

ii- If $\frac{x}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$ then find value of A.

iii- Write partial fraction form $\frac{2x^4-3x^3-4x}{(x^2+2)^2(x+1)^2}$

iv- Define a sequence.

v- Which term of the A.P (with usual notation) $-2, 4, 10, \ldots\ldots$ is 148 ?

vi- Sum the series $(-3) + (-1) + 1 + 3 + 5 + \ldots\ldots\ldots\ldots a_{16}$

vii- Insert three G. Ms between 2 and 32

viii- Find the 12^{th} term of the harmonic sequence $\frac{1}{3}, \frac{2}{9}, \frac{1}{6}, \ldots\ldots\ldots\ldots$

ix- If ${}^nC_8 = {}^nC_{12}$ find n.

x- Find the fifth term of $\left(\frac{3x}{2} - \frac{1}{3x}\right)^{11}$

xi- Use binomial theorem to calculate $(21)^5$ upto three decimal places.

xii- Prove that the result $3^n < n!$ is true for $n = 7, 8$

(Turn over)

4. Write short answers to any NINE questions: **(2 x 9 = 18)**

i- Convert $154^\circ 20''$ to radian measure.

ii- Verify that $\sin^2\frac{\pi}{6} + \sin^2\frac{\pi}{3} + \tan^2\frac{\pi}{4} = 2$

iii- Prove the identity $\frac{\cot^2\theta - 1}{1+\cot^2\theta} = 2\cos^2\theta - 1$

iv- Express $\sin 319^\circ$ as a trigonometric function of an angle of positive degree measure of less than 45°.

v- Show that $\frac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta} = \frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$

vi- If $\cos\alpha = \frac{3}{5}$ then find the value of $\sin 2\alpha$ where $0 < \alpha < \frac{\pi}{2}$

vii- Find the period of $\tan x$.

viii- In the triangle ABC if $c = 16.1$, $\alpha = 42^\circ 45'$ and $\gamma = 74^\circ 32'$. Find a

ix- Define escribed circle.

x- Find the area of triangle ABC if $a = 18$, $b = 24$, $c = 30$

xi- Define inverse sine function.

xii- Solve the equation $\sin 2x = \cos x$ where $x \in [0, 2\pi]$

xiii- Solve $\sin x = -\frac{\sqrt{3}}{2}$ where $x \in [0, 2\pi]$

SECTION II

5- (a) Convert $A \cup (B \cup C) = A \cup (B \cup C)$ into logical form and prove by constructing the truth table. 5

(b) Show that the sum of 'n' A.Ms between 'a' and 'b' is equal to 'n' times their A.M. 5

6- (a) Solve the system of linear equations: 5

$$\begin{aligned} x + 2y + z &= 2 \\ 2x + y + 2z &= -1 \\ 2x + 3y - z &= 9 \end{aligned}$$

(b) Find the number of 6 digit numbers that can be formed from the digits 2, 2, 3, 3, 4, 4. How many of them will lie between 400000 and 430000 ? 5

7- (a) Solve the system of equations: 5

$$x^2 - 5xy + 6y^2 = 0 \quad ; \quad x^2 + y^2 = 45$$

(b) Find the coefficient of x^5 in the expansion of $\left(x^2 - \frac{3}{2x}\right)^{10}$ 5

Prove that $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$ 5

$\frac{2\sin\theta \sin 2\theta}{\cos\theta + \cos 3\theta} = \tan\theta \tan 2\theta$ 5

a triangle are $x^2 + x + 1$, $2x + 1$ and $x^2 - 1$. Prove that the greatest e triangle is 120°. 5

at $\sin^{-1}\frac{77}{85} - \sin^{-1}\frac{3}{5} = \cos^{-1}\frac{15}{17}$ 5

Group-II

ll No.________________

athematics
me: 30 Minutes

(INTER PART-I) 319-(III)
Code: 6196
OBJECTIVE

GROUP: II

PAPER: I
Marks: 20

te: You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling two or more circles will result in zero mark in that question. Attempt as many questions as given in objective type question paper and leave others blank.

1- Expansion of $(1+x)^{\frac{-1}{4}}$ is valid only if

(A) $|x|>1$ (B) $|x|<1$ (C) $|x|<-1$ (D) $|x|>-1$

2- The 8^{th} term of sequence $1,-3,5,-7$ is

(A) 15 (B) -15 (C) 14 (D) -14

3- A reciprocal equation remains unchanged when variable x is replaced by

(A) $-\frac{1}{x}$ (B) $\frac{1}{x}$ (C) $\frac{1}{x^2}$ (D) $-x$

4- The solutions of equation $1+\sin\theta = 0$ are in quadrant

(A) I and IV (B) I and III (C) II and IV (D) III and IV

5- With usual notations, radius r of inscribed circle is given by

(A) $\frac{\Delta}{s}$ (B) $\frac{s}{\Delta}$ (C) $\frac{\Delta}{s-c}$ (D) $\frac{4\Delta}{abc}$

6- If $\tan\theta = \frac{1}{\sqrt{3}}$ and θ is in III quadrant then $\cot\theta$ equals

(A) $\sqrt{3}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

7- ${}^{n-1}C_r + {}^{n-1}C_{r-1}$ equals

(A) ${}^{n+1}C_r$ (B) ${}^{n+1}C_{r+1}$ (C) ${}^{n}C_r$ (D) ${}^{n-1}C_r$

8- $\sin(\cos^{-1}\frac{1}{2})$ equals

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) $\frac{-\sqrt{3}}{2}$ (D) $\frac{-1}{2}$

9- $(x-1)^2 = x^2-2x+1$ is called

(A) equation (B) inequality (C) identity (D) polynomial

10- For any two matrices A and B then $(AB)^t$ equals

(A) AB (B) A^tB^t (C) B^tA^t (D) BA

11- Additive inverse of $a \in \mathbb{R}$ is

(A) 2 (B) 1 (C) $\frac{1}{a}$ (D) $-a$

(Turn over)

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12- With usual notations, the value of $a+b+c$ is

(A) s (B) 2s (C) 3s (D) $\frac{s}{2}$

13- $\cos 315^\circ$ equals

(A) $\tan(-45^\circ)$ (B) $\tan 45^\circ$ (C) $\sin 45^\circ$ (D) $\text{cosec } 45^\circ$

14- If A and B are disjoint then $P(A \cup B)$ equals

(A) $P(A)-P(B)$ (B) $P(A)\,P(B)$ (C) $\frac{P(A)}{P(B)}$ (D) $P(A)+P(B)$

15- If $\begin{bmatrix} \lambda & 4 \\ 3 & 2 \end{bmatrix}$ is singular then λ is equal to

(A) 2 (B) 6 (C) 4 (D) 8

16- The middle term in expansion of $(a+x)^n$ when n is even is

(A) $\left(\frac{n}{2}+1\right)$th term (B) $\left(\frac{n}{2}-1\right)$th term (C) $\left(\frac{n}{2}\right)$th term (D) $\left(\frac{n+1}{2}\right)$th term

17- Period of $\text{cosec } 10x$ is

(A) $\frac{\pi}{10}$ (B) $\frac{2\pi}{5}$ (C) $\frac{\pi}{5}$ (D) $\frac{4\pi}{5}$

18- The domain of relation $f=\{(a, 1), (b, 1), (c, 1)\}$ is

(A) $\{a, b, c\}$ (B) $\{a\}$ (C) $\{b\}$ (D) $\{1\}$

19- If ω is complex cube root of unity then ω^{15} equals

(A) 1 (B) zero (C) ω (D) $-\omega$

20- The arithmetic mean between $\frac{1}{2}$ and $\frac{1}{4}$ is

(A) $\frac{3}{8}$ (B) $\frac{3}{4}$ (C) $\frac{1}{8}$ (D) $-\frac{1}{8}$

Group II

thematics — (INTER PART-I) 319 — GROUP: II — PAPER: I

e: 2:30 hours — **SUBJECTIVE** — Marks: 80

te: Section I is compulsory. Attempt any three (3) questions from Section II.

SECTION I

Write short answers to any EIGHT questions: (2 x 8 = 16)

i- Separate into real and imaginary parts $\frac{i}{1+i}$

ii- Simplify $(i)^{101}$

iii- Show that $\forall z \in c$, $(\bar{z})^2 + z^2$ is a real number.

iv- For the conditional $p \rightarrow q$. Write its inverse and converse.

v- Define disjunction of two statements p and q

vi- If a, b are elements of a group G, then show that $(ab)^{-1} = b^{-1}a^{-1}$

vii- Find x and y if $\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$

viii- Find the value of λ if $A = \begin{bmatrix} 4 & \lambda \\ 7 & 3 \end{bmatrix}$ is singular.

ix- Define upper triangular matrix.

x- Reduce $x^{-2} - 10 = 3x^{-1}$ into quadratic form.

xi- Show that $(x^3 - y^3) = (x-y)(x-\omega y)(x-\omega^2 y)$, where ω is a cube root of unity.

xii- Show that roots of $(p+q)x^2 - px - q = 0$ are rational.

Write short answers to any EIGHT questions: (2 x 8 = 16)

i- Resolve $\frac{7x+25}{(x+3)(x+4)}$ into partial fractions.

ii- Define proper rational fraction.

iii- For the identity $\frac{2x-3}{x(2x+3)(x-1)} = \frac{A}{x} + \frac{B}{2x+3} + \frac{C}{x-1}$ calculate the value of A and C.

iv- Write the first four terms of the sequence $a_n = \frac{n}{2n+1}$

v- How many terms are there in A.P., in which $a_1 = 11$, $a_n = 68$, $d = 3$

vi- Sum the series $\frac{1}{1+\sqrt{x}} + \frac{1}{1-x} + \frac{1}{1-\sqrt{x}} + \dots\dots\dots$ to n terms.

vii- Find the 12^{th} term of the G.P $1+i, 2i, 2(1-i), \dots\dots$

viii- Find the sum of the following infinite geometric series $4 + 2\sqrt{2} + 2 + \sqrt{2} + 1 + \dots\dots$

ix- How many arrangements of the letters of the word 'MATHEMATICS', taken all together, can be made?

x- Prove the formula for n = 1, 2 $\quad 1 + 2 + 4 + \dots\dots + 2^{n-1} = 2^n - 1$

xi- Calculate $(2.02)^4$ by means of binomial theorem.

xii- Expand $(1+x)^{\frac{-1}{3}}$ upto 4-terms, taking the values of x such that the expansion is valid.

(Turn over)

4. Write short answers to any NINE questions: **(2 x 9 = 18)**

i- What is the length of the arc intercepted on a circle of radius 14 cm by the arms of a central angle of 45°?

ii- Evaluate: $\dfrac{1-\tan^2\frac{\pi}{3}}{1+\tan^2\frac{\pi}{3}}$

iii- Prove that: $\dfrac{1-\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1+\sin\theta}$

iv- Prove that: $\tan\left(\frac{\pi}{4}-\theta\right)+\tan\left(\frac{3\pi}{4}+\theta\right) = 0$

v- Prove that: $\tan 2a = \dfrac{2\tan\alpha}{1-\tan^2\alpha}$

vi- Find the value of $\cos 2\alpha$ when $\sin\alpha = \frac{12}{13}$ where $0 < \alpha < \frac{\pi}{2}$

vii- Find the period of $\tan\frac{x}{3}$

viii- State law of cosines.

ix- Find the area of the triangle ABC, given three sides $a = 524$, $b = 276$, $c = 315$

x- Show that: $r_1 = s\tan\frac{\alpha}{2}$

xi- Prove that: $\sin^{-1}x = \frac{\pi}{2} - \cos^{-1}x$

xii- Find the solution of equation: $\sin x = \dfrac{-\sqrt{3}}{2}$

xiii- Solve the equation: $\sin^2 x + \cos x = 1$

SECTION II

5- (a) Prove that all 2 x 2 non-singular matrices over the real field form a non-abelian group under multiplication. 5

(b) For what value of n, $\dfrac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the positive geometric mean between a and b? 5

6- (a) Use Cramer's rule to solve the system: 5

$$\begin{aligned} 3x_1 + x_2 - x_3 &= -4 \\ x_1 + x_2 - 2x_3 &= -4 \\ -x_1 + 2x_2 - x_3 &= 1 \end{aligned}$$

(b) The members of a club are 12 boys and 8 girls. In how many ways can a committee of 3 boys and 5 girls be formed? 5

7- (a) Solve $4.2^{2x+1} - 9.2^x + 1 = 0$ 5

(b) Find the term involving a^4 in the expansion of $\left(\frac{2}{x} - a\right)^9$ 5

8- (a) Prove that: $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$ 5

(b) Reduce $\sin^4\theta$ to an expression involving function of multiple of θ raised to the first power. 5

9- (a) The sides of a triangle are x^2+x+1, $2x+1$, x^2-1. Prove that the greatest angle of the triangle is 120°. 5

(b) Prove that: $\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5} - \tan^{-1}\frac{8}{19} = \frac{\pi}{4}$ 5